ECE 344

MICROWAVE FUNDAMENTALS PART1-Lecture 7

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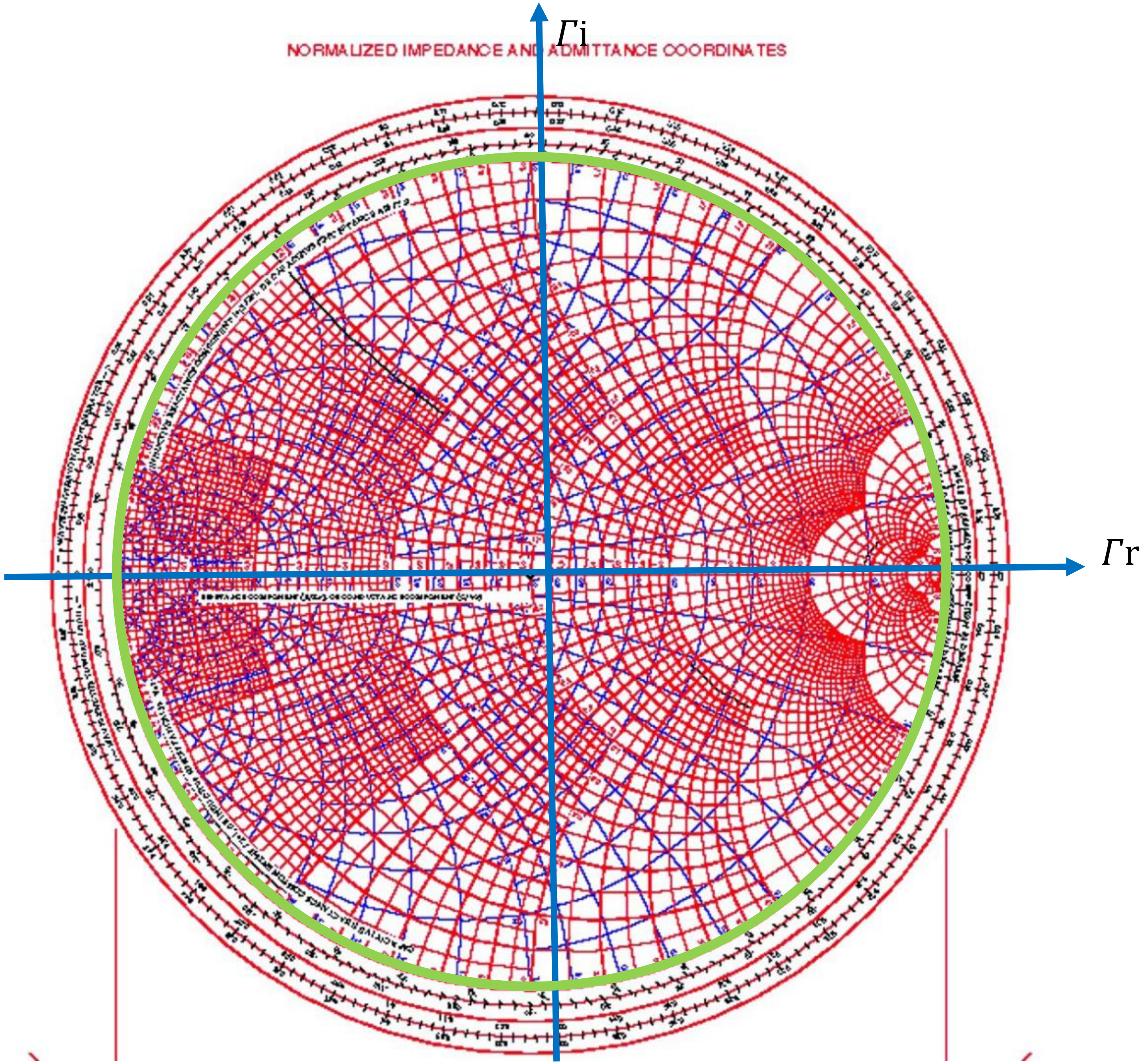
Many Slides from: ECE 5317_6351 Microwave Engineering Prof. David R. Jackson

Impedance/Admittance smith ch

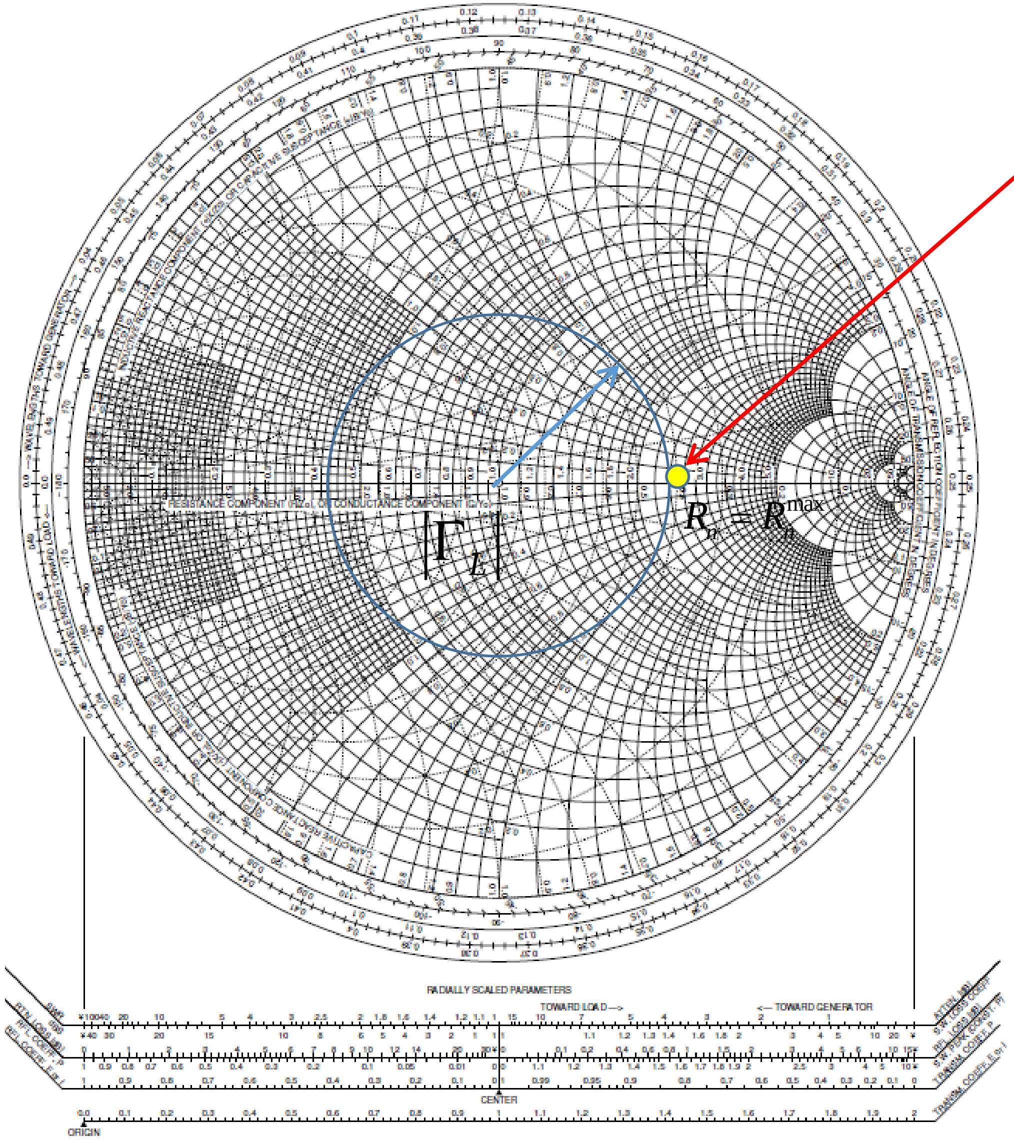
A chart of Γ

$\Gamma = \Gamma r + j\, \Gamma i$

Max reflection circle
$|\Gamma|=1$



Standing Wave Ratio



The SWR is given by the value of R_n on the positive real axis of the Smith chart.

Proof:

$$SWR = \frac{1+|\Gamma_L|}{1-|\Gamma_L|}$$

$$\Rightarrow \; R_n^{\max} = \frac{1+|\Gamma_L|}{1-|\Gamma_L|}$$

Impedance/Admittance smith chart

You will learn

- Locate impedance on smith chart read corresponding admittance and vise versa, read Γload, move along TL read corresponding Γin, Zin, VSWR, lmin, lmax
- Quarter wave transformation
- Adding elements (series-shunt) to load impedance on Smith chart
- Find input impedance to an arbitrary circuit (may contain series, shunt, TL connections)

Impedance (Z) Chart

$$Z(-\ell) = Z_0\left(\frac{1+\Gamma}{1-\Gamma}\right) \qquad \Gamma = \Gamma(-\ell)$$

$$Z_n(-\ell) \equiv \frac{Z(-\ell)}{Z_0} = \left(\frac{1+\Gamma}{1-\Gamma}\right)$$

Define

$$Z_n = R_n + jX_n \;\; ; \;\; \Gamma = \Gamma_R + j\Gamma_I$$

Substitute into above expression for $Z_n(-\ell)$:

$$R_n + jX_n = \left(\frac{1+(\Gamma_R + j\Gamma_I)}{1-(\Gamma_R + j\Gamma_I)}\right)$$

Next, multiply both sides by the RHS denominator term and equate real and imaginary parts. Then solve the resulting equations for Γ_R and Γ_I in terms of R_n and X_n. This gives two equations.

Impedance (Z) Chart (cont.)

1) Equation #1:

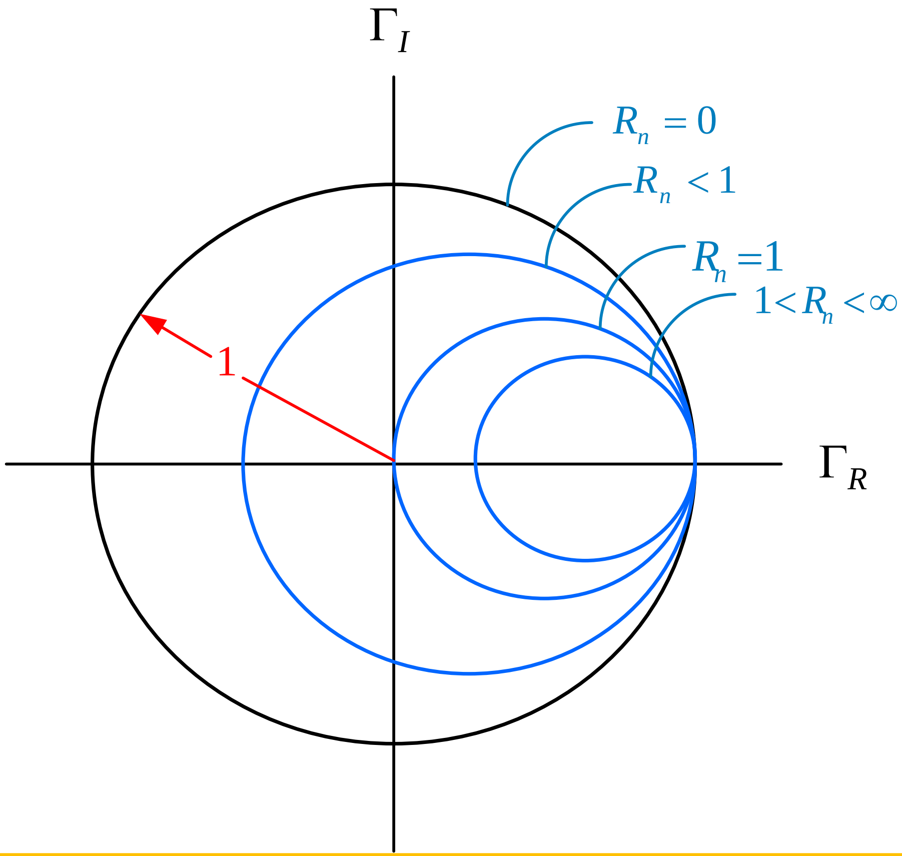
$$\left(\Gamma_R - \frac{R_L}{1+R_L}\right)^2 + \Gamma_I^2 = \left(\frac{1}{1+R_L}\right)^2$$

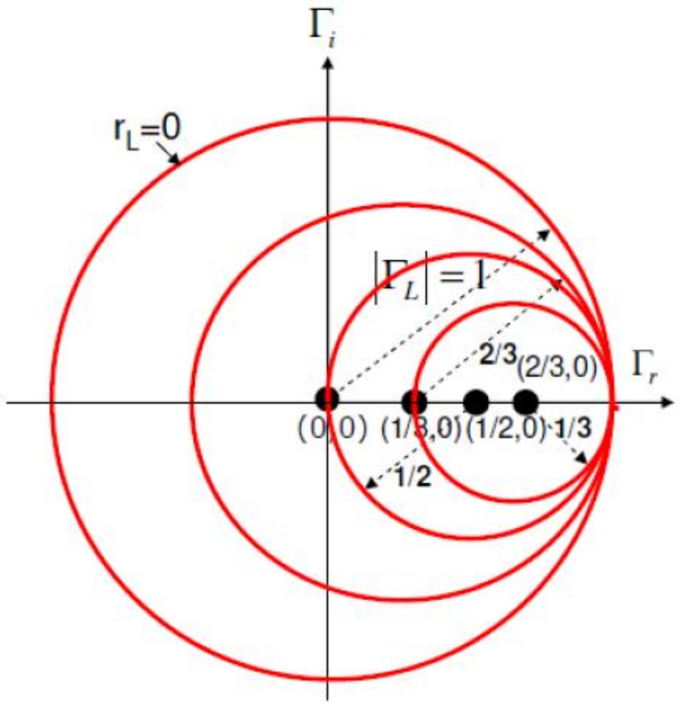
$$\text{center} = \left(\frac{R_L}{1+R_L}, 0\right)$$

$$\text{radius} = \frac{1}{1+R_L}$$

Transforming "r"

r	Radius	Center
0	1	(0,0)
1/2	2/3	(1/3,0)
1	1/2	(1/2,0)
2	1/3	(2/3,0)
∞	0	(1,0)

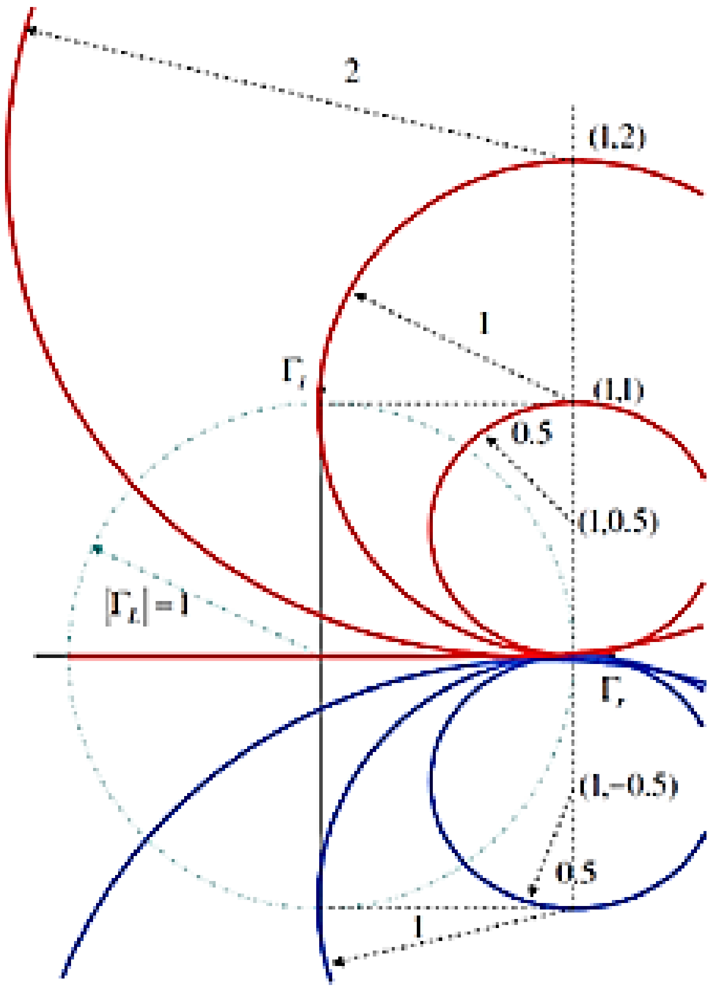




Impedance (Z) Chart (cont.)

2) Equation #2: $$(\Gamma_R - 1)^2 + \left(\Gamma_I - \frac{1}{X_n}\right)^2 = \left(\frac{1}{X_n}\right)^2$$

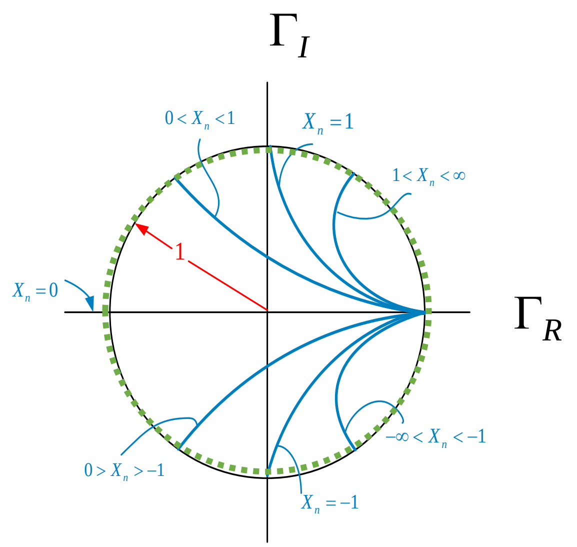
center $= \left(1, \frac{1}{X_n}\right)$ radius $= \frac{1}{|X_n|}$



Transforming "x"

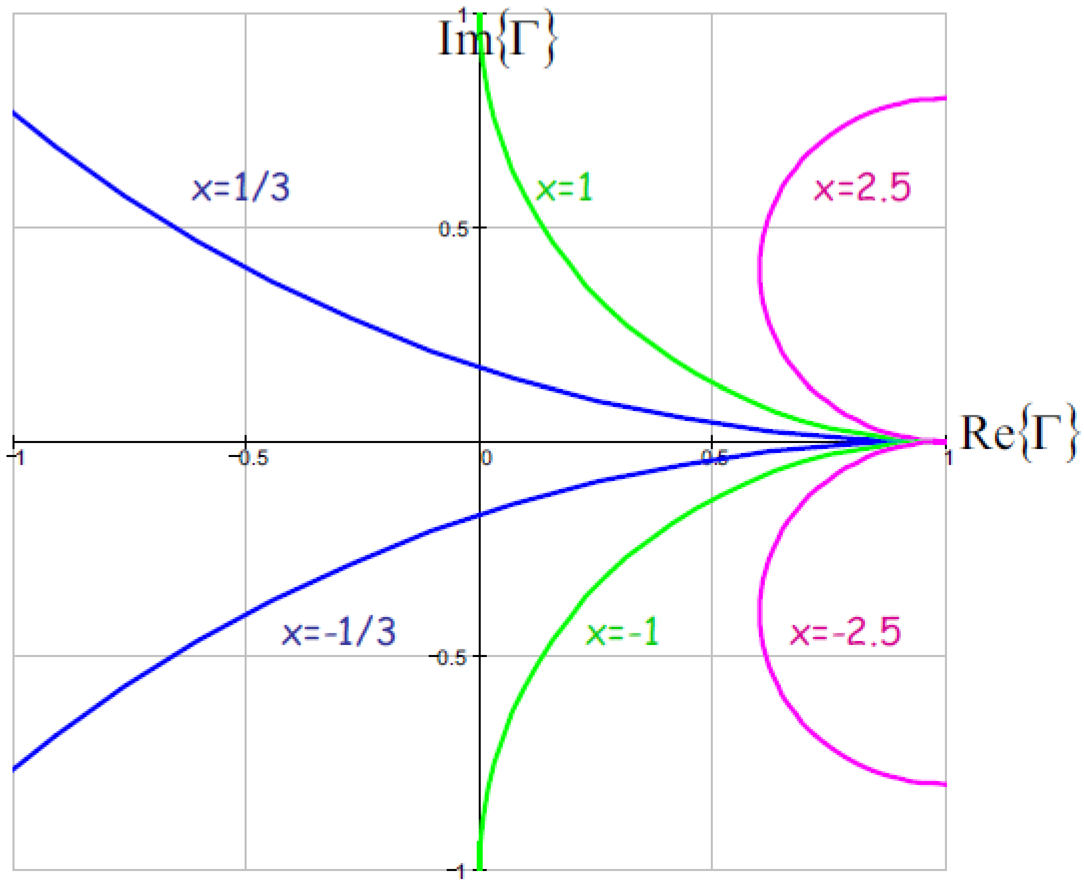
x	Radius	Center
0	∞	(1,∞)
0.5	2	(1,2)
1	1	(1,1)
2	0.5	(1,0.5)
∞	0	(1,0)

x	Radius	Center
0	∞	(1,-∞)
-0.5	2	(1,-2)
-1	1	(1,-1)
-2	0.5	(1,-0.5)
-∞	0	(1,0)

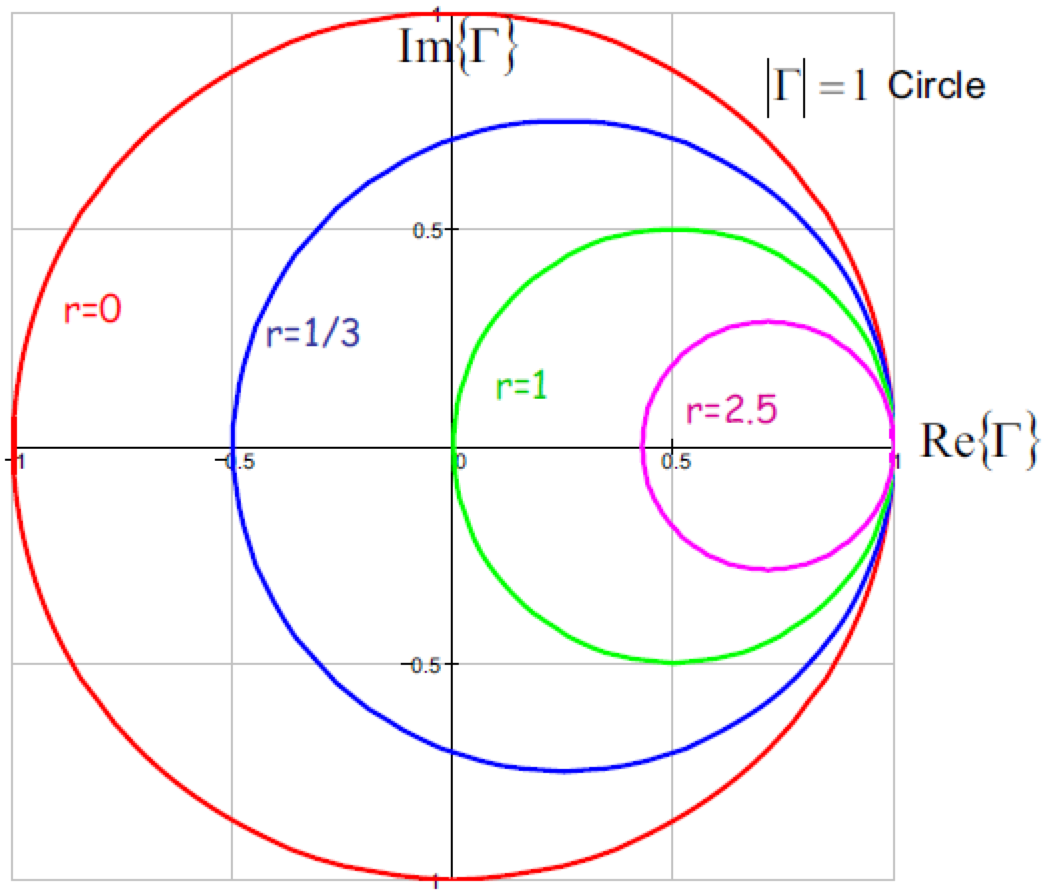


Impedance Smith Chart

Smith Chart – Imaginary Circles



Smith Chart – Real Circles



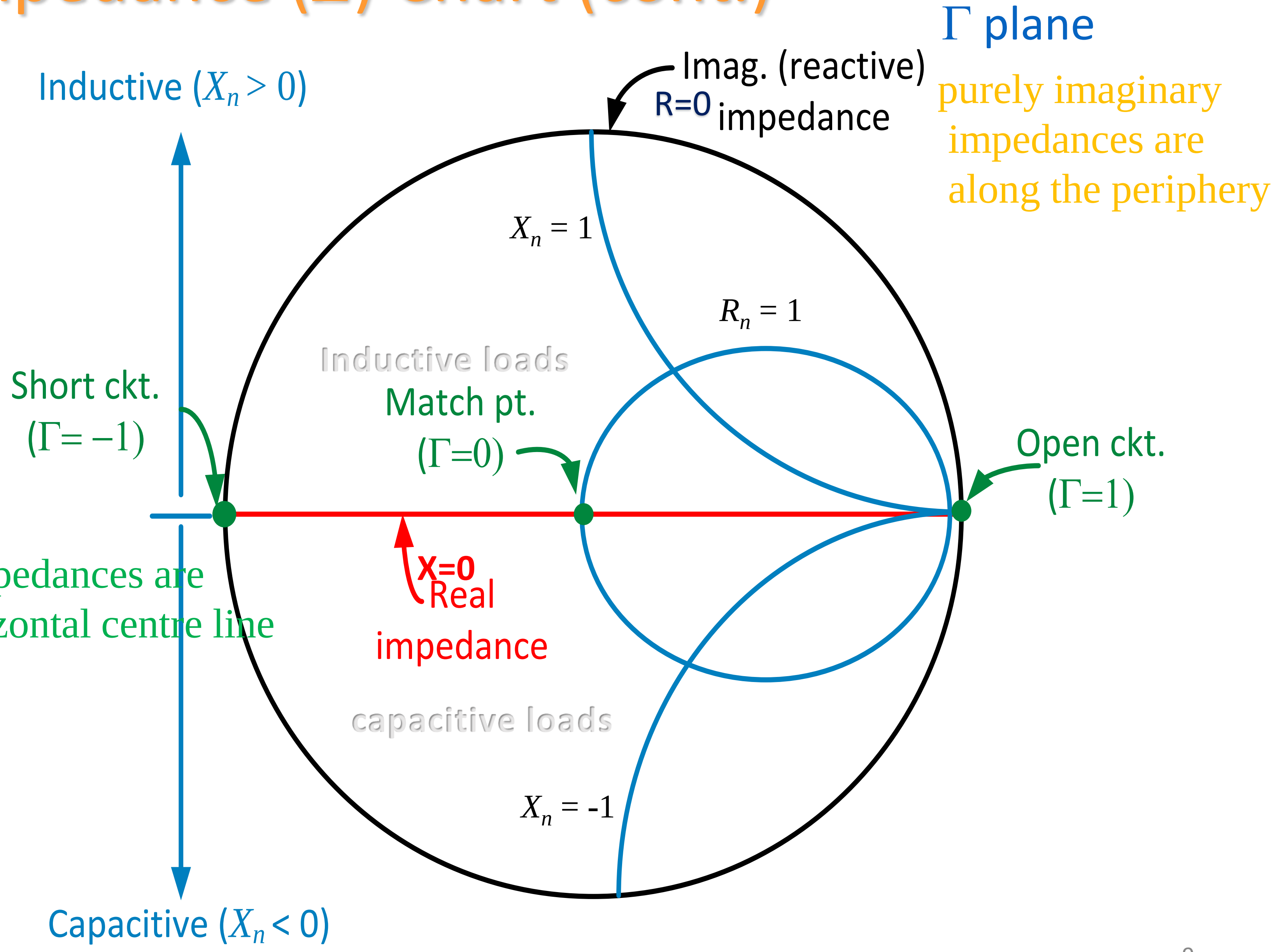
Impedance (Z) Chart (cont.)

Important Points:

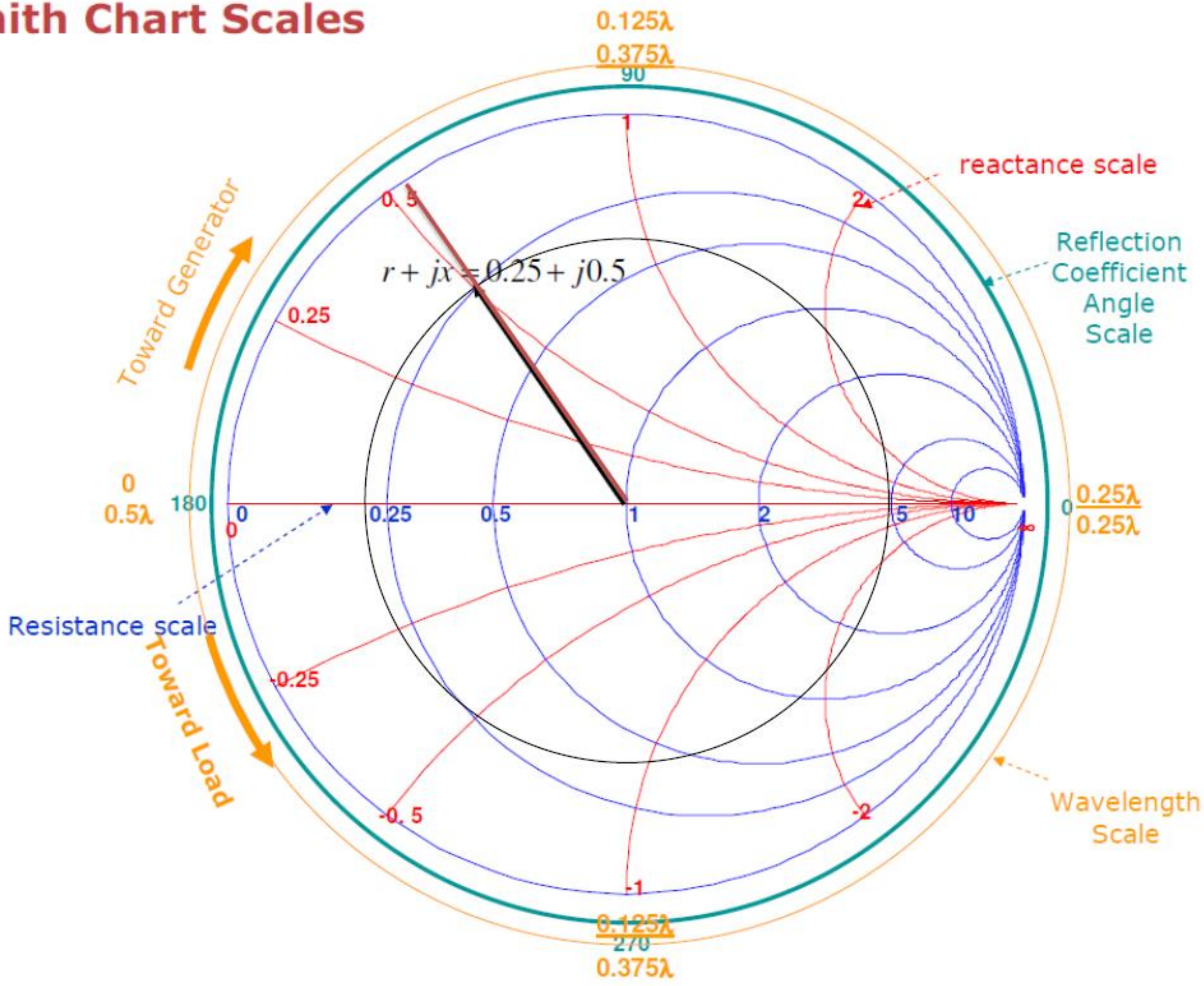
- *Short Circuit*
  $\Gamma = -1,\ z = 0$
- *Open Circuit*
  $\Gamma = 1,\ z \to \infty$
- *Matched Load*
  $\Gamma = 0,\ z = 1$

- *The circle $|\Gamma| = 1$ describes a **lossless element** (C or L)*

purely real impedances are along the horizontal centre line

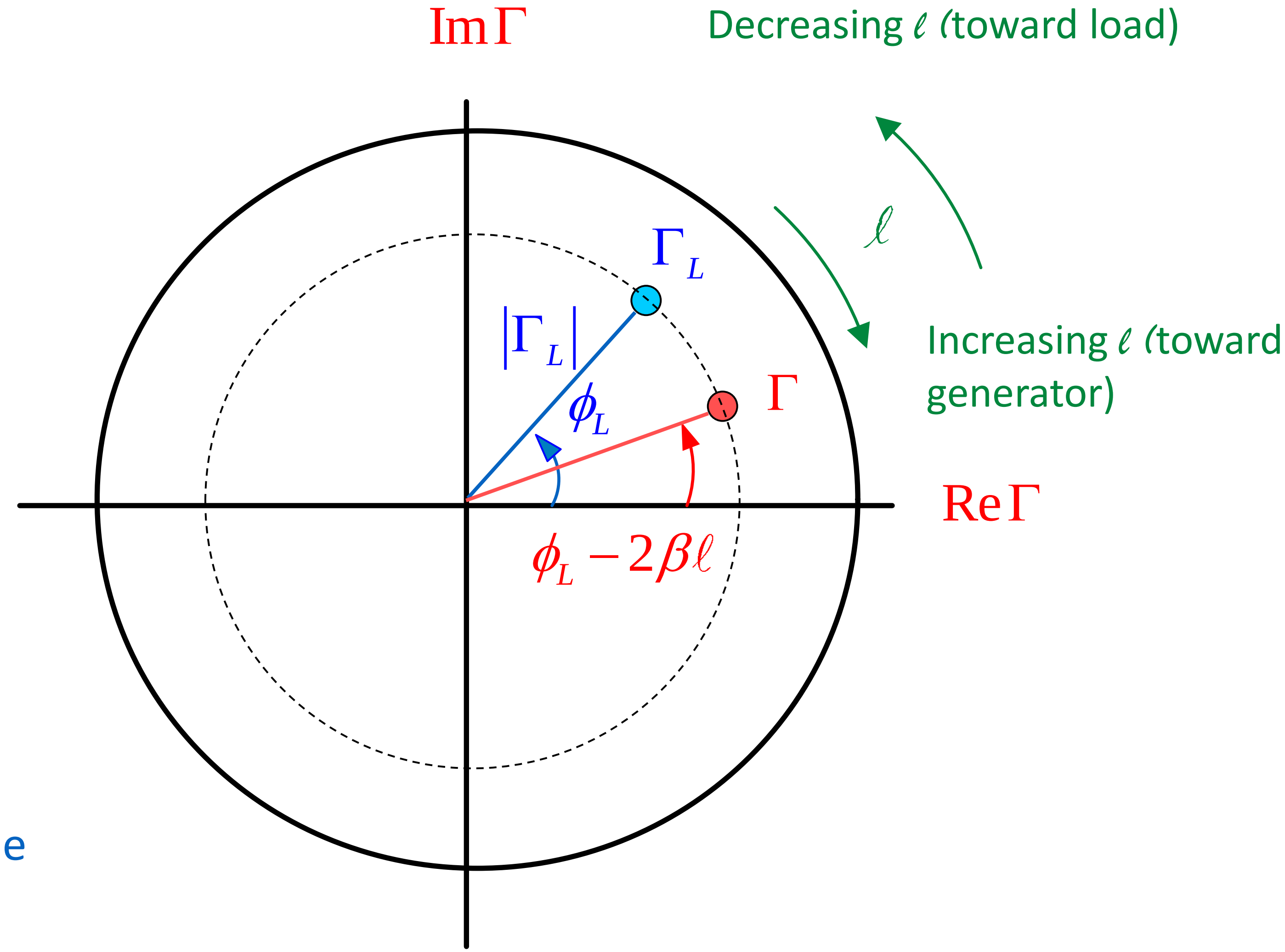


Smith Chart Scales



Complex Γ Plane

$$
\begin{aligned}
\Gamma &= \Gamma(-\ell) \\
&= \Gamma_R + j\Gamma_I \\
&= \Gamma_L e^{j(-2\beta\ell)} \\
&= |\Gamma_L| e^{j(\phi_L - 2\beta\ell)}
\end{aligned}
$$



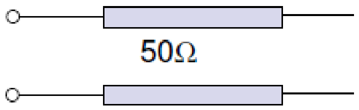
$$\Gamma(-\ell) = |\Gamma_L| e^{j(\phi_L - 2\beta\ell)}$$

Lossless line

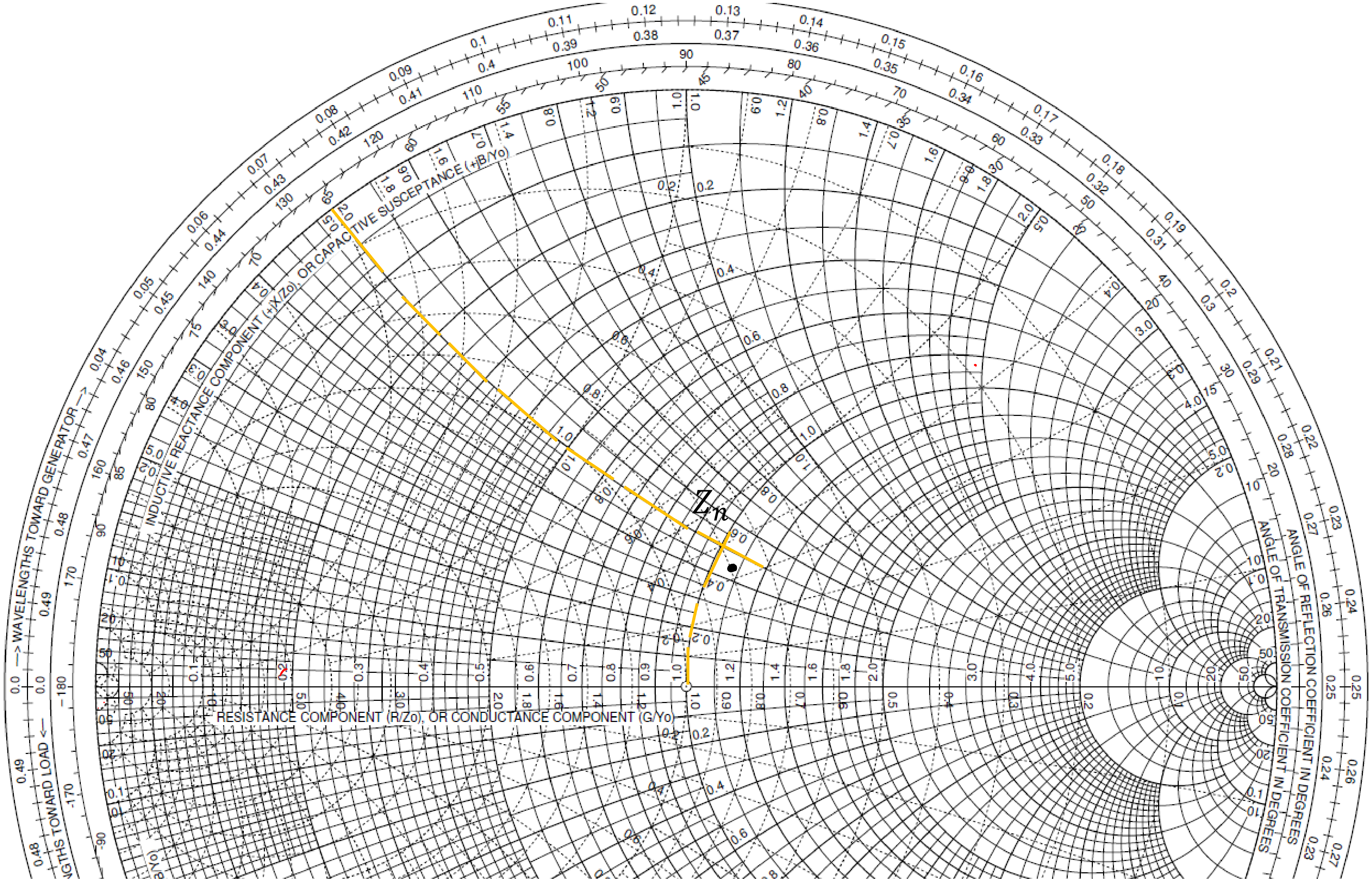
Example 1

Locate Z

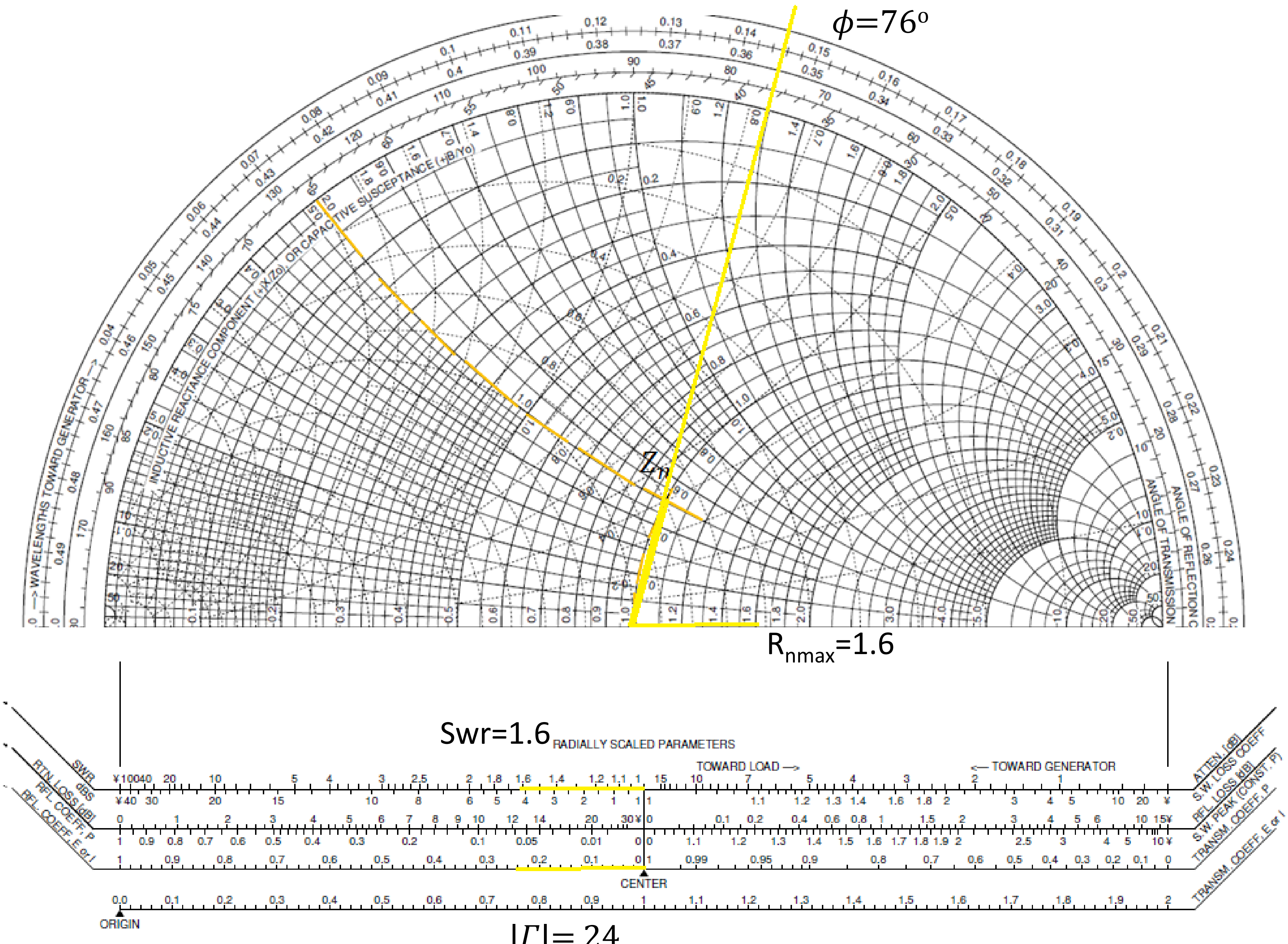
e.g. $Z_L = 50 + j\,25$



$$Z_n = 1 + j0.5$$

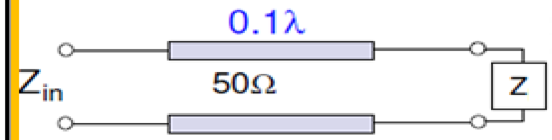


READ :
SWR,
Γ



Move along TL from load toward Generator

e.g. $Z_L = 50 + j\,25\ \Omega$



All 50 Ω, constant ρ

$Z_{in} = 50(1.65 + j\,0.1)\ \Omega$

$\Gamma_{in} = 0.24\ (4^\circ)$

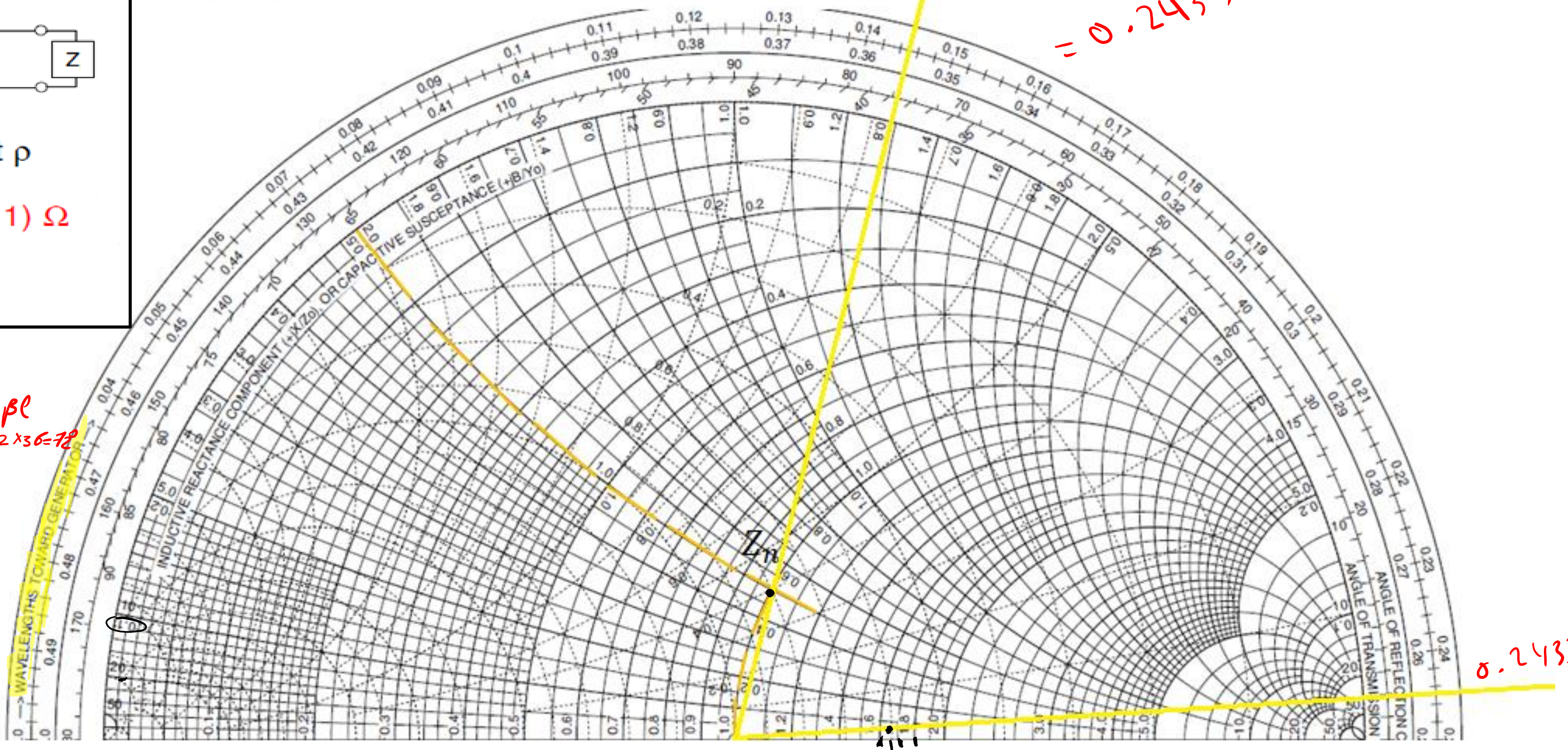
By calculation

$\Gamma_{in} = |\Gamma| \angle \phi - 2\beta l$, $2\beta l = 2\times 36 = 72$

$= 0.24 \angle 76^\circ - 72^\circ$

$= 0.24 \angle 4^\circ$

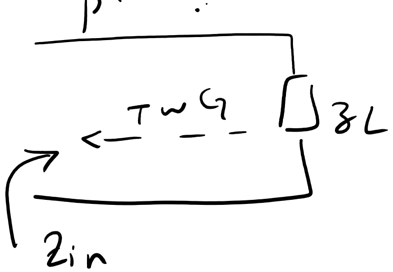
So use wave length scale directly instead of calc.



Example2: Read Distance to load

$z_L = 1.5 + j\,1.5$

$z_{in} = 0.65 - j$



$$\beta l = \frac{2\pi}{\lambda}(0.36 - 0.194)\lambda$$

$$\beta l = 2\pi \times 0.166 = 59.76 \simeq 60^\circ$$

By Equations :-

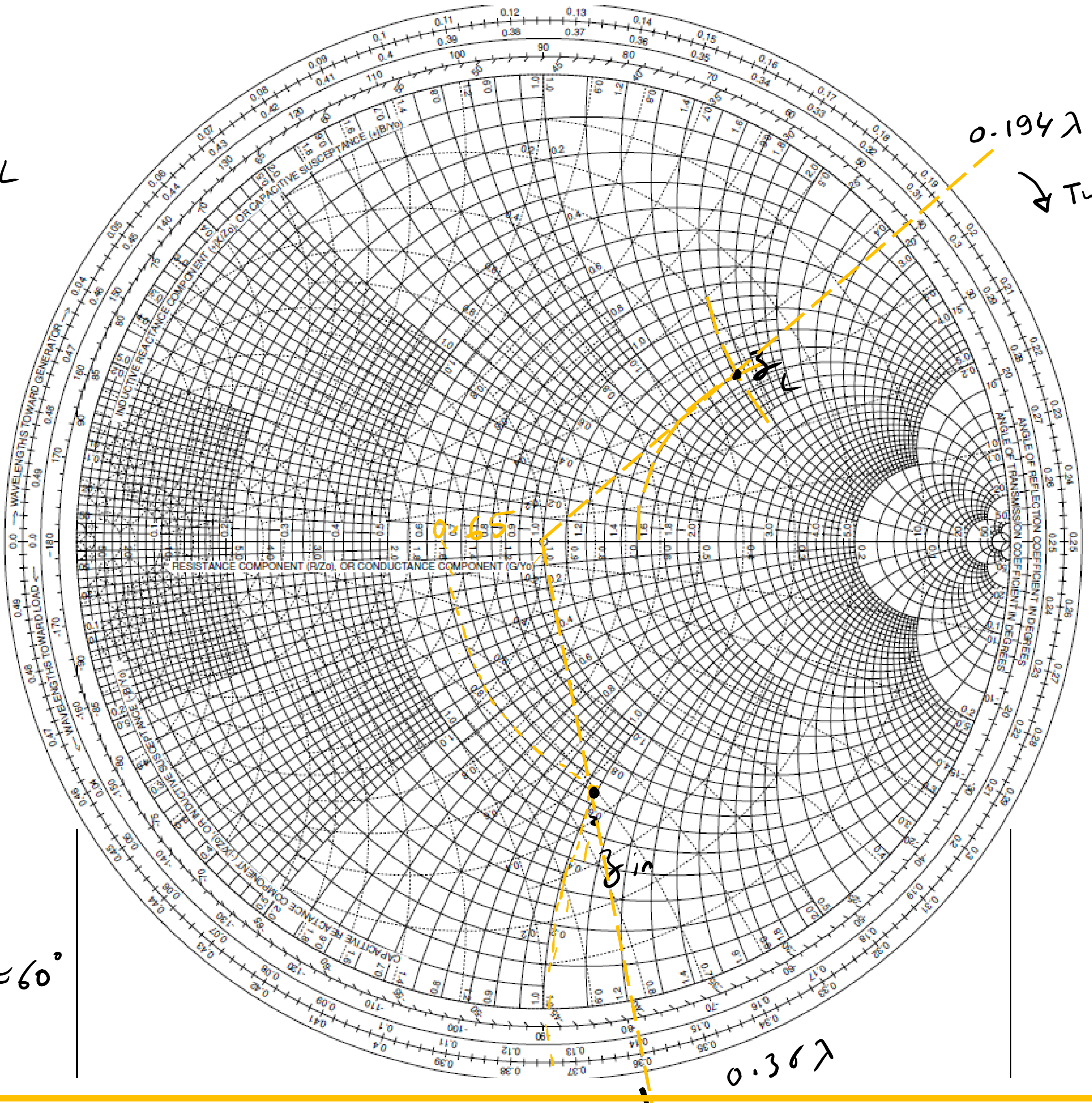
for $z_L = 1.5 + j\,1.5 \rightarrow \Gamma_L = \frac{z_L - 1}{z_L + 1} = 0.54\angle 40.6$

for $z_{in} = 0.65 - j \rightarrow \Gamma_{in} = 0.54\angle -78$

$= |\Gamma_L|\angle \phi - 2\beta l$

$\therefore\ 40.6 - 2\beta l = -78$

$$\beta l = \frac{78 + 40.6}{2} = 59.3 \simeq 60^\circ$$



Example 3(2.2 in the book)

Z_0=100, z_L= 0.4+j0.7
(Zl=40+j70)
l=.3λ
Find Γ_L ,Γ(-*l*), Zin,
SWR, Return loss

From smith chart
We can find:
From reflection
coefficient E scale
| Γ_L |=.59.θ=104°
From SWR s
SWR=3.8
From return
Scale, return
loss=4.6dB
,Γ(-*l*)=.59∠−